Ejercicios de Derivadas Resueltos

Halla la función derivada de estas funciones:

26 **a)** $f(x) = \dfrac{e^x + e^{-x}}{2}$ **b)** $f(x) = (x^2 - 3)^3$

a) $f'(x) = \dfrac{e^x - e^{-x}}{2}$ b) $f'(x) = 6x(x^2 - 3)^2$

27 **a)** $f(x) = \dfrac{x^3 - x^2}{x^2}$ **b)** $f(x) = \sqrt{x^2 + 1}$

a) $f'(x) = 1$ (si $x \neq 0$) b) $f'(x) = \dfrac{x}{\sqrt{x^2 + 1}}$

28 **a)** $f(x) = \sqrt[3]{(x + 6)^2}$ **b)** $f(x) = \sqrt{sen\ x}$

a) $f'(x) = \dfrac{2}{3\sqrt[3]{(x + 6)}}$ b) $f'(x) = \dfrac{cos\ x}{2\sqrt{sen\ x}}$

29 **a)** $f(x) = \dfrac{-3}{\sqrt{1 - x^2}}$ **b)** $f(x) = 7^{x+1} \cdot e^{-x}$

a) $f'(x) = -3(1 - x^2)^{-1/2}$; $f'(x) = \dfrac{3}{2}(1 - x^2)^{-3/2} \cdot (-2x) = \dfrac{-3x}{\sqrt{(1 - x^2)^3}}$

b) $f'(x) = 7^{x+1} \cdot ln\ 7 \cdot e^{-x} + 7^{x+1} \cdot e^{-x} \cdot (-1) = 7^{x+1} \cdot e^{-x}\ (ln\ 7 - 1)$

30 **a)** $f(x) = \dfrac{1}{3x} + \dfrac{x}{3}$ **b)** $f(x) = ln\ 3x + e^{\sqrt{x}}$

a) $f'(x) = \dfrac{-1}{3x^2} + \dfrac{1}{3}$ b) $f'(x) = \dfrac{3}{3x} + e^{\sqrt{x}}\ \dfrac{1}{2\sqrt{x}} = \dfrac{1}{x} + \dfrac{e^{\sqrt{x}}}{2\sqrt{x}}$

31 **a)** $f(x) = \left(\dfrac{x}{1 + x^2}\right)^2$ **b)** $f(x) = e^{2x} \cdot tg\ x$

a) $f'(x) = 2\left(\dfrac{x}{1 + x^2}\right) \cdot \dfrac{1 + x^2 - x \cdot 2x}{(1 + x^2)^2} = \dfrac{2x}{(1 + x^2)} \cdot \dfrac{1 - x^2}{(1 + x^2)^2} = \dfrac{2x\ (1 - x^2)}{(1 + x^2)^3}$

b) $f'(x) = 2e^{2x}\ tg\ x + e^{2x}\ (1 + tg^2\ x) = e^{2x}\ (2\ tg\ x + 1 + tg^2\ x) = e^{2x}\ (1 + tg\ x)^2$

32 **a)** $f(x) = \dfrac{x^3}{(x - 1)^2}$ **b)** $f(x) = cos^2\ x + e^{sen\ x}$

a) $f'(x) = \dfrac{3x^2(x - 1)^2 - x^3 \cdot 2(x - 1)}{(x - 1)^4} = \dfrac{3x^2(x - 1) - 2x^3}{(x - 1)^3} = \dfrac{3x^3 - 3x^2 - 2x^3}{(x - 1)^3} =$

$= \dfrac{x^3 - 3x^2}{(x - 1)^3}$

b) $f'(x) = 2\ cos\ x\ (-sen\ x) + e^{sen\ x} \cdot cos\ x = cos\ x\ (-2\ sen\ x + e^{sen\ x})$

33 **a)** $f(x) = \sqrt{\dfrac{x^3}{x^2-4}}$ **b)** $f(x) = \left(\dfrac{x}{2}\right)^3 \cdot e^{1-x}$

a) $f(x) = \left(\dfrac{x^3}{x^2-4}\right)^{1/2} \to f'(x) = \dfrac{1}{2}\left(\dfrac{x^3}{x^2-4}\right)^{-1/2} \cdot \dfrac{3x^2(x^2-4) - x^3 \cdot 2x}{(x^2-4)^2} =$

$= \dfrac{1}{2}\left(\dfrac{x^2-4}{x^3}\right)^{1/2} \cdot \dfrac{3x^4 - 12x^2 - 2x^4}{(x^2-4)^2} = \dfrac{1}{2} \cdot \dfrac{1}{\sqrt{x^3}} \cdot \dfrac{x^4-12x^2}{\sqrt{(x^2-4)^3}} =$

$= \dfrac{x^4-12x^2}{2\sqrt{x^3(x^2-4)}}$

b) $f'(x) = 3\left(\dfrac{x}{2}\right)^2 \dfrac{1}{2} \cdot e^{1-x} + \left(\dfrac{x}{2}\right)^3 \cdot e^{1-x} \cdot (-1) = \dfrac{3}{8}x^2 e^{1-x} - \dfrac{1}{8}x^3 e^{1-x} =$

$= \dfrac{x^2}{8} e^{1-x}(3-x) = \dfrac{x^2(3-x)\, e^{1-x}}{8}$

34 **a)** $f(x) = sen\ \dfrac{3\pi}{2}$ **b)** $f(x) = log\ \dfrac{x^2}{3-x}$

a) $f'(x) = 0$

b) $f(x) = log\, x^2 - log(3-x) = 2\, log\, x - log(3-x)$

$f'(x) = \dfrac{2}{x\, ln\, 10} + \dfrac{1}{(3-x)\, ln\, 10}$

35 **a)** $f(x) = tg^3\, x^2$ **b)** $f(x) = \sqrt{ln\, x}$

a) $f'(x) = 3\, tg^2\, x^2 (1 + tg^2\, x^2) \cdot 2x = 6x\, tg^2\, x^2\, (1 + tg^2\, x^2)$

b) $f'(x) = \dfrac{1}{2x\sqrt{ln\, x}}$

36 **a)** $f(x) = arc\ sen\ \dfrac{x^2}{3}$ **b)** $f(x) = arc\ tg\,(x^2+1)$

a) $f'(x) = \dfrac{1}{\sqrt{1-(x^2/3)^2}} \cdot \dfrac{2x}{3} = \dfrac{2x/3}{\sqrt{1-x^4/9}} = \dfrac{2x}{\sqrt{9-x^4}}$

b) $f'(x) = \dfrac{1}{1+(x^2+1)^2} \cdot 2x = \dfrac{2x}{1+(x^2+1)^2}$

37 **a)** $f(x) = arc\ cos\ \dfrac{1}{x}$ **b)** $f(x) = arc\ tg\ \dfrac{\sqrt{x}}{2}$

a) $f'(x) = \dfrac{-1}{\sqrt{1-(1/x)^2}} \cdot \dfrac{-1}{x^2} = \dfrac{1/x^2}{\sqrt{1-1/x^2}} = \dfrac{1}{x\sqrt{x^2-1}}$

b) $f'(x) = \dfrac{1}{1+\left(\sqrt{x}/2\right)^2} \cdot \dfrac{1}{4\sqrt{x}} = \dfrac{1}{4\sqrt{x}\,(1+(x/4))} = \dfrac{1}{\sqrt{x}\,(4+x)}$

38 **a)** $f(x) = \sqrt{arc\ tg\ x}$ **b)** $f(x) = arc\ cos\ e^{-x}$

a) $f'(x) = \dfrac{1}{2\sqrt{arc\ tg\ x}} \cdot \dfrac{1}{(1 + x^2)} = \dfrac{1}{2(1 + x^2)\sqrt{arc\ tg\ x}}$

b) $f'(x) = \dfrac{-1}{\sqrt{1 - e^{-2x}}} \cdot e^{-x} \cdot (-1) = \dfrac{e^{-x}}{\sqrt{1 - e^{-2x}}}$

39 **a)** $f(x) = \sqrt{x + \sqrt{x}}$ **b)** $f(x) = arc\ tg\left(\dfrac{1 - x}{1 + x}\right)$

a) $f'(x) = \dfrac{1}{2\sqrt{x + \sqrt{x}}} \cdot \left(1 + \dfrac{1}{2\sqrt{x}}\right) = \dfrac{1}{2\sqrt{x + \sqrt{x}}} \cdot \left(\dfrac{2\sqrt{x} + 1}{2\sqrt{x}}\right) =$

$= \dfrac{2\sqrt{x} + 1}{4\sqrt{x} \cdot \sqrt{x + \sqrt{x}}} = \dfrac{2\sqrt{x} + 1}{4\sqrt{x(x + \sqrt{x})}} = \dfrac{2\sqrt{x} + 1}{4\sqrt{x^2 + x\sqrt{x}}}$

b) $f'(x) = \dfrac{1}{1 + [(1 - x)/(1 + x)]^2} \cdot \dfrac{-1(1 + x) - (1 - x)}{(1 + x)^2} =$

$= \dfrac{1}{1 + [(1 - x)^2/(1 + x)^2]} \cdot \dfrac{-1 - x - 1 + x}{(1 + x)^2} =$

$= \dfrac{(1 + x)^2}{(1 + x)^2 + (1 - x)^2} \cdot \dfrac{-2}{(1 + x)^2} = \dfrac{-2}{(1 + x)^2 + (1 - x)^2} =$

$= \dfrac{-2}{1 + x^2 + 2x + 1 + x^2 - 2x} = \dfrac{-2}{2x^2 + 2} = \dfrac{-2}{2(x^2 + 1)} = \dfrac{-1}{x^2 + 1}$